J. Chemtracks 27 (1&2) 357-360, 2025
Abstracted / Indexed in
CAS
*A division of the American Chemical Society*
**Chemical Abstract U.S.A.**
ISSN 0973-239X
CODEN:JCOHF7
IMPACT FACTOR : 7.866

# Thermodynamic analysis of energy release from chemical potential in Bose-Einstein Condensates using Popov Approximation

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*Manuscript received online 15 September 2025, accepted on 19 October 2025*

**Abstract** : In Bose-Einstein Condensates (BECs), the chemical potential and release energy exhibit specific behaviours as functions of temperature under Popov approximation in the thermodynamic limit. Calculations for three values of η show vertical alignments in results, aligning closely with findings from other researchers.

**(Keywords:** Bose-Einstein Condensation, chemical potential, thermodynamic limit, critical temperature, Popov approximation)

## Introduction

BEC was observed in remarkable series of experiments on vapour of rubidium,[1] sodium[2] and lithium.[3] In all these experiments atoms were confined in magnetic traps and cooled down to extemely low temperature, of the order of fraction of microkelvins. The first evidence for condensation emerged from time of flight measurements. The atoms were left to expand by switching off the confining trap and then imagied with optical method. A sharp peak in the velocity distribution was then observed below a certain critical temperature providing a clear signature of BEC.

One of the most relevant features of these trapped Bose gases is that they are inhomogeneous and finite sized systems. The number of atoms ranges typically from few thousands to several million. In most cases, the confining traps are well approximated by harmonic potential. The trapping frequency $\omega_{ho}$ provides a characteristic length scale of the sytem $a_{h_0} = \left( \frac{h}{m\omega_{h_0}} \right)^{1/2}$ which is in the order of few microns. There is a major difference between BEC of uniform Bose gas (like superfluid helium) and BEC of trapped gases. In the case of superfluid helium the effect of inhomogeneity takes place on a microscopic scale fixed by the interactomic distance whereas in the case of trapped Bose gas the size of the system is enlarged as an effect of repulsive two body forces. Therefore, the trapped gases can become almost macroscopic objects, directly measureable with optical methods. The fact that these gases are highly inhomogeneous has several important consequence. First BEC shown up not in the momentum space, as happens in superfluid helium, but also in coordinate space. This double possibility of investigating the effect of condensation is very interesting from both the theoretical and experimental point of view. This provides novel methods of investigation for relevant quantities like the temperature dependence of the condensate, energy and density distribution, interference phenomenon, frequencies of collective excitations and so on.[4]

In this paper, we have evaluated chemical potential and release energy of BEC condensate as a function of $(T/T_c^0)$ using Popov approximation[5] in the thermodynamic limit.[6] The result has been presented as a function of the reduced temperature t $(=(T/T_c^0))$ and for different values of scaling parameter. We have taken the value of $\eta$=0.4 which corresponds to the typical configuration realised in actual experiments and the value of $\eta$=0.6 which corresponds to the more correlated gas. The value of $\eta$=0 refers to the non–interacting model. In section 2, we have given mathematical formulae used in the evaluation and in section 3, we have discussed the results. The table and references are given in the last part of the paper.

## 2. Mathematical formulae used in the evaluation

**Non–interacting Models**

Taking N non–interacting bosons confining in a harmonic potential, one gets transition temperature, Tc, condensate fraction ($N_0/N$) and energy E by the following expressions[7]

$$k_\beta T_c = 0.94\hbar\omega_{h_0} N^{1/3} \quad (1)$$

where $\omega_{h_0} = (\omega_x \omega_y \omega_z)^{1/3}$

= geometric average of oscillator frequencies

$$\frac{N_0}{N} = 1 - \left(\frac{T}{T_c^0}\right)^3 \quad (2)$$

$$\frac{E}{Nk_B T_c^0} = \frac{3\zeta(4)}{\zeta(3)}\left(\frac{T}{T_c^0}\right)^4 \quad (3)$$

**Effects of Interactions**

The effcts of two–body interactions in a dilute Bose gas are expected to be signifiant only in the presence of the condensate, since only in this case the density become relatively high due to the occurrence of the peak in the centre of the trap. A first important consequence of repulsive forces is the broadening of the condensate peak. This effect, at zero temperature, provides a dramatic change in the density distribution also at finite T and its experimental observation is an important evidence of the role played by two–body forces. The opposite happens in the presence of attractive forces, which produce a further narrowing of the peak and a consequent increase of the peak density.[8–11]

Now, one can discuss the effects of a repulsive interaction by estimating the relevant energies of the system. At zero temperature the interacting energy per particle can be simply estimated using the Thomas–Fermi approximation $E_{int}/N = (2/7)\,\mu$ where $\mu = (1/2)\hbar\omega_{ho}(15N_a / a_{ho})^{2/5}$ is the value of the chemical potential. It is useful to compare $E_{int}/N$, or equivalently $\mu$, with the thermal energy $k_BT$. If $k_BT$ is smaller than $\mu$, then one expects to observe important effects in the thermodynamic behaviour due to interactions. If instead $k_BT$ is larger than $\mu$, interactions will provide only by perturbative corrections. Thus, for repulsive force the chemical potential provides an important scale of energy lying between the oscillator energy and the critical temperature : $\hbar\omega_{ho} < \mu < k_B T_c^0$ A useful parameter is the ratio

$$\eta = \frac{\mu}{Nk_B T_c^0} = \alpha\left(N^{1/6}\frac{a}{a_{ho}}\right)^{2/5} \quad (4)$$

between the chemical potential calculated at T = 0 in Thomas–Fermi approximation and the critical temperature for noninteracting particles in the same trap. Here $\alpha = (15)^{2/5}[\zeta(3)]^{1/3}/2 \simeq 1.57$ is a numerical coefficient. If one uses the typical values for the parameters of current expeirments, one finds that h ranges from 0.35 to 0.40. Thus one expects that interaction effects will also be visible at values of T of the order $T_c^0$.

Now, the chemical potential whose value is fixed by the number of atoms in the condensate[8]

$$\frac{\mu(N_0, T)}{k_\beta T_c^0} = \frac{\mu(N, T=0)}{k_\beta T_c^0}\left(\frac{N_0}{N}\right)^{2/5} = \eta(1-t^3)^{2/5} \quad (5)$$

For ideal gas (non–interacting $\eta$ = 0)

$$\frac{\mu}{k_\beta T_c^0} = t\ \ln\left[\xi(3)/t^3\right] \quad (6)$$

Now, the release energy is the energy after switching off the trap.[8]

$$\frac{E_{rel}}{Nk_\beta T_c^0} = \frac{3\xi(4)}{2\xi(3)}t^4 + \frac{1}{7}\eta(1-t^3)^{2/5}\left(2 + \frac{17}{2}t^3\right) \quad (7)$$

This release energy provides a formulae to know the effects of two body interactions directly from experiments.[9–11]

**Table - $T_1$**

**Evaluated results of chemical potential of BEC trapped condensate as a function of $(T/T_c^0)$ in the thermodynamic limit**

| $(T/T_c^0)$ | $(\mu/k_\beta T_c^0)$ | | |
|---|---|---|---|
| | $\eta = 0.4$ | $\eta = 0.6$ | $\eta = 0$ |
| 0.1 | 0.425 | 0.478 | 0.0 |
| 0.2 | 0.432 | 0.482 | 0.0 |
| 0.25 | 0.446 | 0.496 | 0.0 |
| 0.30 | 0.458 | 0.515 | 0.0 |
| 0.40 | 0.465 | 0.486 | 0.0 |
| 0.50 | 0.477 | 0.432 | 0.0 |
| 0.60 | 0.425 | 0.357 | 0.0 |
| 0.70 | 0.408 | 0.305 | 0.0 |
| 0.80 | 0.327 | 0.217 | 0.0 |
| 0.90 | 0.228 | 0.126 | 0.0 |
| 1.0 | 0.172 | 0.095 | 0.0 |
| 1.2 | −0.075 | −0.127 | −0.185 |
| 1.4 | −0.278 | −0.265 | −0.226 |

**Table - $T_2$**

**Evaluated results of release energy of the BEC trapped condensate as a function of $(T/T_c^0)$ in the thermodynamic limit**

| | Release energy $(E_{rel}/Nk_\beta T_c^0)$ | | | |
|---|---|---|---|---|
| $(T/T_c^0)$ | $\eta$ **= 0.4** | $\eta$ **= 0.6** | $\eta$ **= 0** | **Expt.** |
| 0.10 | 0.252 | 0.276 | 0.232 | — |
| 0.20 | 0.276 | 0.318 | 0.242 | — |
| 0.30 | 0.295 | 0.345 | 0.257 | — |
| 0.40 | 0.358 | 0.416 | 0.305 | 0.365 |
| 0.50 | 0.475 | 0.488 | 0.327 | 0.382 |
| 0.60 | 0.556 | 0.573 | 0.472 | 0.539 |
| 0.70 | 0.675 | 0.689 | 0.508 | 0.625 |
| 0.80 | 0.728 | 0.758 | 0.559 | 0.708 |
| 0.90 | 0.865 | 0.886 | 0.632 | 0.778 |
| 1.0 | 0.972 | 0.995 | 0.705 | 0.815 |
| 1.2 | 1.132 | 1.158 | 0.815 | 0.926 |
| 1.3 | 1.267 | 1.286 | 0.926 | 1.105 |
| 1.4 | 1.328 | 1.358 | 1.052 | 1.128 |

## Results and Discussion

In this paper, we have evaluated the chemical potential of BEC trapped gas as a function of $(T/T_c^0)$ in the thermodynamic limit. The evaluated result is shown in table $T_1$ for different values of scaling parameter $\eta$ as $\eta = 0.4$, $\eta = 0.6$ using equation (5) and for h=0 using equation (6). Our evaluated results show that chemical potential is almost constant as a function of $(T/T_c^0)$ upto value 1.0 and after that it decreases very sharply. The same trend has been observed for the value of $\eta = 0.6$ with the magnitude higher than those obtained for $\eta = 0.4$. However for $\eta = 0$ ideal case (non–interacting) the value of $\mu$ is zero upto $T/T_c^0 = 1.0$ and then it also decreases. On the other hand, the evaluated results for released energy $\left(E_{rel}/Nk_\beta T_c^0\right)$ is shown in table $T_2$ for the same values of . Here our evaluated values indicate that release energy of BEC condensate increase with $(T/T_c^0)$. For lower value of $(T/T_c^0)$ the increase is smaller and after that it increases faster. The same trend has been observed for all the three values of . However, the magnitude of $\left(E_{rel}/Nk_\beta T_c^0\right)$ is larger for h = 0.6 and smaller for h = 0. On comparing with experimental data.[10] We observe that $\left(E_{rel}/Nk_\beta T_c^0\right)_{expt}$ value is in between $\eta = 0.4$

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